Distance Fields in Visualization and Graphics

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Abstract

We report on distance fields, i.e. 2D or 3D arrays which hold signed/unsigned distance to objects/features of interest Such fields are a powerful tool for the accomplishment of different tasks in 2D/3D image processing and analysis, computer vision, visualization and graphics.

1 Distance Transforms

Brute force distance calculations are very expensive, since for each voxel of the field the distance to the nearest surface point has to be evaluated by inspecting all objects of the scene. Satherley and Jones [SJ01] reported days long calculations on high-end workstations. Approximate techniques have been therefore developed, cumulatively known as distance transforms [RP66], which try to estimate the Euclidean distance in a reasonable time. Their main idea is to replace the global distance computation by a local propagation of distances in a small neighborhood. These approaches require several passes through the data.

Chamfer distance transforms, proposed by Borgefors [Bor86], issue from an assumption that the distance can be computed only from values at neighboring positions plus a mask constant. This approach enables precise computation the *city-block* (L_1 metrics) and *chessboard* (L_∞ metrics) distances, but can only approximate the Euclidean distance (L_2 metrics). Therefore, more precise albeit slower *Vector distance transforms* were introduced [Dan80], propagating also position of the nearest surface point. To speed up the computation, the multipass mask propagation was later replaced by region growing [Cui97] and level-set [Set99, BMW98] approaches.

Traditionally, the distance computation issues from a segmented binary data, where the object surface voxel positions are confined to fixed grid point coordinates. However, mostly in volume graphics applications, this proved to be a precision limitation (for example, in offset surface computation [BMW98, SJ01]) and therefore techniques were developed, propagating distances to surfaces defined with subvoxel precision. To compute the field with even higher precision, for each voxel a corresponding nearest surface point is recomputed, utilizing information stored with its already processed neighbors.

2 Application Areas

Distance transforms are a powerful tool for the accomplishment of different tasks in 2D/3D image processing and analysis, computer vision, visualization and graphics [PT92]. 2D fields registering a signed distance to object contours were used to interpolate surfaces of the segmented 3D objects in tomographic data [RU90, HZB92, JM94]. In [SB93] the technique was extended even for unsegmented gray-scale data. Brummer et at in [BMEL93] used the distance fields to estimate a probability of the brain tissue presence in the detection of brain contours in MRI data. The distance fields were further used to create bounding spheres for the collision detection in robotics [GS00], to build a skeletal representation of objects in colonoscopy and angioscopy [ZKT98, ZT99, BSB$^+$00, BKS01], to define a cost function in the registration of volumetric data sets [Bor88] and to flatten complex surfaces (a human colon) by means of a curvilinear ray casting [BWKG01].

Distances and distance fields play a key role in volume graphics, namely in object representation, object-to-objects metamorphosis techniques and in acceleration of volume data rendering.

2.1 Representation of Objects by Distance Fields

The early voxelization techniques represented geometric objects in volumetric grids only by means of binary values: one value was selected for the object or its surface and the other one for the background [KS86]. Although such kind of representation is completely suitable for many applications, it is not precise enough for high fidelity surface rendering. The nature of the problem resides in that in binary voxelization a discontinuous (and therefore with unbounded frequency spectrum) inside-outside function representing the object is sampled with a finite step. The natural solution to the problem seems to be the lowpass filtering of the inside-outside function, introduced in the *Volume-sampled* voxelization technique by Wang and Kaufman [WK93, WK94]. This approach significantly improved the appearance of the renditions of the voxelized objects, but still, sume problems remained: (i) the object details were smoothed out (manifested by a shift of the reconstructed surface in the convex and concave surface areas), and (ii) the gradient was reconstructed with an up to several degrees high error.

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An alternative technique [Šrá94a, ŠK98], which resides in registration of the distance to the object surface, eliminates the aforementioned problems. Of course, there are certain limits of its application to small objects and high curvature surfaces, as it is with all techniques working in the discrete space, but this new technique is still up to two orders of magnitude more precise than the filtering one.

The distance fields were later used for the object representation by several authors. Jones [Jon96] voxelized and subsequently rendered triangular meshes. Gibson [Gib98] showed how the distance fields can be used to smooth out surfaces in the segmented tomographic data by means of elastic surface nets. Breen et al [BMW98, BM99] used the distance fields to construct offset surfaces for superellipsoid models and to morph different geometric model types (polygonal meshes, CSG models and tomographic scans) in a single animation [DBM01]. A great potential of the distance field representation has been recently shown in volume sculpting, where significant steps toward creation of the so-called *digital clay* were performed. Traditional modeling and sculpting tools, based on the surface representation (polygonal or parametric patches) suffer from limitations given by the representation, as, for example, insufficient versatility and unintuitive user interface. These drawbacks are eliminated if the object surfaces are represented by the distance field isosurfaces (adaptive fields [Fri01], two level hierarchies [Bær02]), due to their unconstrained deformation ability and a possibility to implement intuitive sculpting operations (for example, cutting, carving, sawing, spraying).

The problem of the discrete space representation of objects is that only details with certain minimal dimensions can be represented. One way how to decrease this minimal dimension for the given volume resolution is to register a modified distance profile instead of the plain distance itself. In [ŠK99] *erfc* of the signed distance is used, which together with a suitable reconstruction filter enables to decrease to about one half the volume resolution while keeping the same quality of the details. Another approach, issuing from the hierarchical representation of the distance volume, was presented by Frisken et al [FPRJ00] and Bærentzen [Bær02]. Here, the continuous distance field is hierarchically sampled, until a certain homogeneity limit or maximal resolution is reached. This approach, although more algorithmicaly and computationally complex than representation by regular grids, this approach enables to represent simultaneously objects with significantly different dimensions.

2.2 Accelerated Ray-Tracing of Volumetric Data

In ray tracing complex scenes most of the processor time is spent on the ray-object intersection tests [Whi80]. Numerous acceleration techniques were therefore proposed with the aim to minimize the number of the tests by excluding from the consideration beforehand all the objects for which such test fails. One category of such techniques employs uniform subdivision of the scene space in voxels [FTI86], each with a list of relevant objects assigned. The voxels pierced by the ray are then inspected in the direction of the ray progress until the first intersection is found.

Ray tracing is a popular rendering technique also in volume visualization due to its algorithmic simplicity and versatility: within the same framework one can implement different rendering methods (direct surface and volume rendering, MIP) both in software and hardware [Pfi00, MDH$^+$01]. Object representation by the distance fields is similar to the aforementioned uniform subdivision, with two basic differences: instead of a list of objects, a single volume primitive is assigned to each grid location and the number of voxels is typically several orders of magnitude higher (512^3 and often even more). In such a case, even traversal of the empty background voxels surrounding the objects can be time consuming.

One possibility how to minimize the time spent for the traversal of the voxels pierced by the ray in large grids is to identify the empty background voxels by segmentation and to gather them in macro-regions, which can be then safely skipped. Devillers [Dev89] proposed to build overlapping cuboid regions and to assign each background voxel to one of them. Other authors used octrees to build hierarchical nonoverlapping macro regions [Lev90, SW91].

Distances for the ray traversal speed-up were for the first time used by Zuiderveld et al [ZKV92]. After the data segmentation, a distance to the object surface is assigned to each background voxel by a distance transform [Bor86]. This information is then utilized during the ray traversal by adapting the sampling step accordingly. The authors used this RADC (Ray Acceleration by Distance Coding) scheme for their implementation of the direct volume rendering by compositing. A similar *proximity clouds* technique was used by Cohen and Sheffer [CS94] to render surfaces from voxelized data. They observed an uncertainty in the sense that some important non-background voxels were skipped either. Therefore, in order to eliminate this drawback, they proposed to decrease the distances by 1 and to switch the ray traversal algorithm to a 6-connected line generator in the object vicinity.

Both the RADC and proximity clouds techniques can work with different discrete approximations of the Euclidean distance. This in not the case of the *Chessboard Distance* (CD) voxel traversal technique [Šrá94b], which issues exclusively from the chessboard distance. The CD transform defines a cubic macro-region around each voxel with its sides aligned with the voxel faces. The simple cubic geometry enables further optimizations and even an extension of the technique to rectilinear grids with variable voxel dimensions [ŠK00].

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