

Computation and visualization of magnetic fields

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Abstract

The computation and visualization of 3D magnetic fields are time and resource-consuming tasks, especially if there are several magnets in the space. In our program system there is a possibility to place a number of different cylindrical magnets of different sizes in different positions. The generated magnetic field is computed and presented. Furthermore, the program system is able to find the optimal configuration of magnets with respect to a user defined function, for example maximal magnetic induction. The final aim of the system is to give a tool for planning magnetic configurations to be used in medical applications.

Keywords: computation of magnetic field, interactive visualization, optimization

1 Introduction

There are several programs [1, 2, 3] for the computation of magnetic fields in 2D or 3D. For example, the system ViziMag [1] works only in 2D. Other programs, for example [3], simulate electromagnetic fields and visualize dynamic fields, and there are also programs specialized for planning electric circuits [4].

Our program handles 3D magnetic fields generated by static cylindrical magnets. We apply elliptic integrals for computing the electromagnetic fields of permanent magnets. The generated field is computed by numerical approximations. The program is able to generate output files for other applications, for example, the output can be a binary file suitable for 3D Slicer [5]. Furthermore, it contains an optimizer also, which is able to determine the optimal configuration of magnets in order to achieve the best possible solution for a user-required function or restraint on such a function.

The application area of such a system can be the medicine. Nowadays evidence based medicine applies analgesics basically in drug form with undesirable side effects. An alternative solution has lately come up applying static magnetic field to reduce pain [6]. Due to the magnetic field, the stimulus of the nerve-cells, the sensation of the pain (the impulse of the forwarded stimulus to the brain) can be reduced. The mechanism of

action is based on the altered behavior of opioid receptors under an external magnetic field [7]. If we can construct a magnetic field, which blocks the stimulus in a given part of the living tissue, we can substitute present medical solutions.

2 Concepts

In this chapter we discuss the way of operation of our system. It is described how to compute the magnetic field and then the database for representing the magnets, the optimization program, and finally the user interface of our system are discussed.

2.1. Computation of Magnetic Field

The space is a three dimensional (3D) array consisting of magnetic induction vectors. For modeling in a realistic way, precise computations have to be done in discrete points of the space, because the magnitude of the vector coordinates of the magnetic field induction can change in a wide range between $1 \cdot 10^{-12}$ T. The program simulates the permanent magnets by electromagnets, that is, it calculates the given magnet field as if it would be generated by current carrying loops. The calculation of the fields is reduced to the computation of the fields of the current carrying loops. This calculation is done in a series of steps. First, we compute the magnetic field of one current carrying loop in a plane section (Figure 1, 2). It is calculated with elliptic integrals [8] based on the Biot-Savart law.

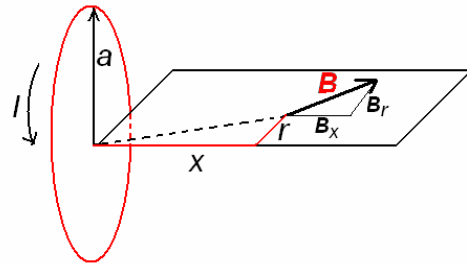


Figure 1: Notation used in the calculation

Accordingly, the magnetic induction on the x-axis and in radial directions is as follows:

$$\mathbf{B}_x = \mathbf{B}_0 \frac{1}{\pi \sqrt{Q}} \left[E(k) \frac{1 - \alpha^2 - \beta^2}{Q - 4\alpha} + K(k) \right],$$

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$$\mathbf{B}_r = \mathbf{B}_0 \frac{\gamma}{\pi \sqrt{Q}} \left[E(k) \frac{1 + \alpha^2 + \beta^2}{Q - 4\alpha} - K(k) \right],$$

where

\mathbf{B} (tesla)	magnetic induction vector (sum of two field components)
\mathbf{B}_x	magnetic induction (coil axis)
\mathbf{B}_r	magnetic induction (radial direction)
I (ampere)	current in the wire
a (meter)	radius
x (meter)	distance on the axis from the center
r (meter)	radial distance from the axis
$K(k)$	complete elliptic integral of the first kind
$E(k)$	complete elliptic integral of the second kind
μ_0	permeability of vacuum

$$\alpha = \frac{r}{a}, \quad \beta = \frac{x}{a}, \quad \gamma = \frac{x}{r}, \quad k = \sqrt{\frac{4\alpha}{Q}},$$

$$Q = [(1 + \alpha)^2 + \beta^2], \quad \mathbf{B}_0 = i\mu_0 / 2a$$

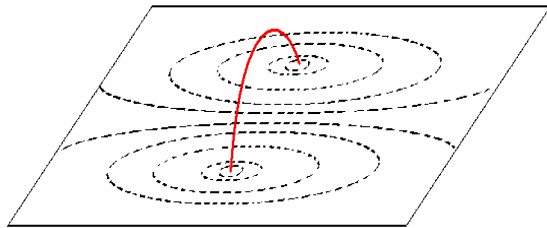


Figure 2: Magnetic field of a current carrying loop in a plane section

Then we rotate (Figure 3) and mirror (Figure 4) this plane, because the magnetic field is symmetrical. Now we get the complete three-dimensional (3D) space of the current carrying loop.

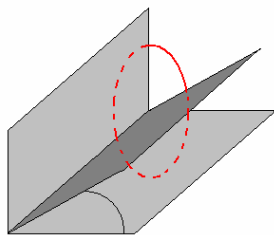


Figure 3: Get a 3D subspace by rotating the plane

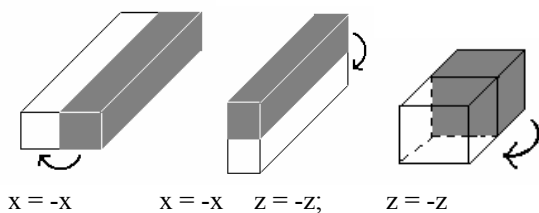


Figure 4: Calculating the 3D magnetic field space of a current carrying loop by mirroring

Next, we construct a cylindrical magnet by superposing the magnet fields shifted according to the different shifts of the same current carrying loop (Figure 5). We can use the principle of superposition effectively without re-computation of the fields of shifted magnets. In fact, we approximate the permanent magnet in this way with a number of electromagnets (current carrying loops).

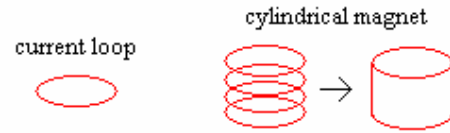


Figure 5: A cylindrical magnet as represented by a group of current carrying loops

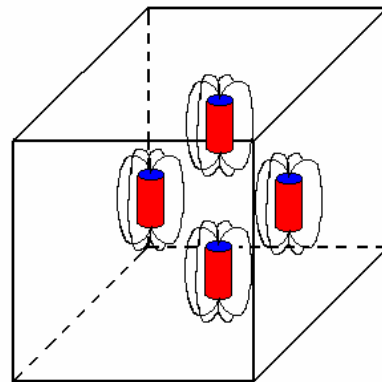


Figure 6: The 3D magnetic field is the sum of the four base spaces generated by the four magnets

Based on this idea the program calculates the cubic fields of each magnet only once and the saved (cubic) magnetic fields can be used later. The actual field of the given configuration of the magnet is generated as the sum of the suitable cubic shaped spaces, considered here as base spaces (Figure 6, Figure 10).

We compare the computation results of our program with the program written in Matlab by Szabó [10]. The results are identical although that program uses a different approximation to compute the magnetic fields.

2.2. Data Base for Storing the Magnets

The principle of the storage is the following. The parameters of the magnets (for example sizes and positions) are one component of the data in our system. Another data element represents the magnetic field. The data of magnetic fields and the magnets are collected in different lists. Actually, the list of magnets is a list of lists, because its elements are lists, which consist of magnets with the same parameters. Every magnet list connected with a magnetic field list, so the storage is more effective (Figure 7).

Of course, the magnetic spaces, which are not used at a given moment can be removed from their list in order

to avoid using too much memory. This opportunity is important (see Table 1).

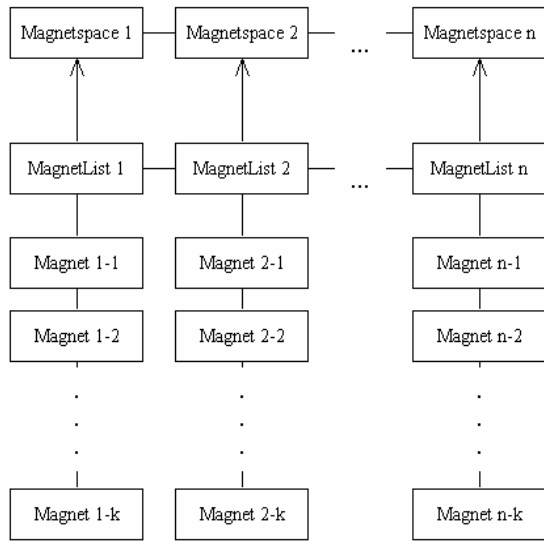


Figure 7: The implemented data structure according to the principle of storing the magnets

2.3. Optimization

The goal of the optimization is to construct an optimal configuration which generates, for example, the largest magnetic induction gradient in the given space (the target function can be given by the user). In this special case the aim is to have a magnetic field such that the sum of the absolute values of the gradients in the space is maximal. We used the numerical optimizer Powell's method, which was implemented in Numerical Recipes [9]. It searches the optimum starting from an initial configuration (for example coordinates of centre, radius, current) The Powell's method examines the small modifications in the present configuration and chose the best one. It continues this procedure until an optimum point is found (or it reaches a limiting number of operations). It needs a special function which is called target function. This will be optimized by the maximum magnetic gradient.

2.4. Data Saving and Loading

The computed field of the magnets can be saved as a "raw" dataflow in one coherent form or as transversal cross-sections of the space. The later solution is useful if we want to cooperate with other programs, for example, with 3D Slicer (used in medical image processing). With the help of the 3D Slicer we can visualize the saved magnetic fields easily (Figure 14).

The induction represented on the given plane of the magnetic field can be saved in different picture formats (for example in png, bmp, jpeg, pgm). In this case the picture represents the logarithmic normalized image of the induction values on the given cross-section (Figure 8, Figure 9) Naturally, the program visualizes the induction, but due to the standard image file formats, these pictures

can be seen with standard image viewers without our program.

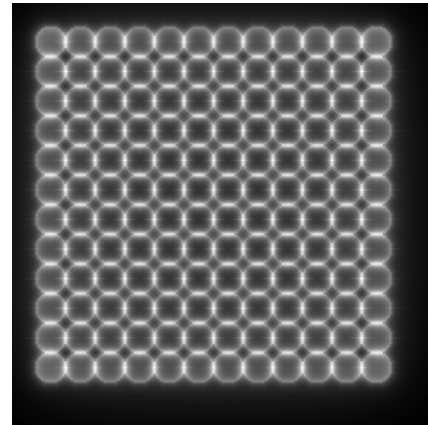


Figure 8: Arrangement of cylindrical magnets in a matrix with dimensions 12x12 (top view of a grayscale induction image, high-resolution)

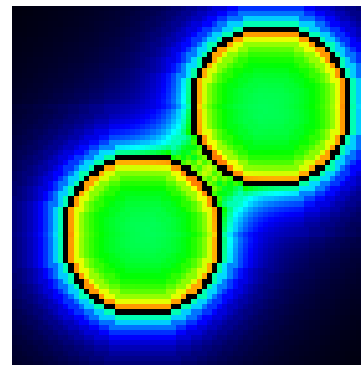


Figure 9: The magnetic field induced by two magnets (top view of a colored induction image, low-resolution)

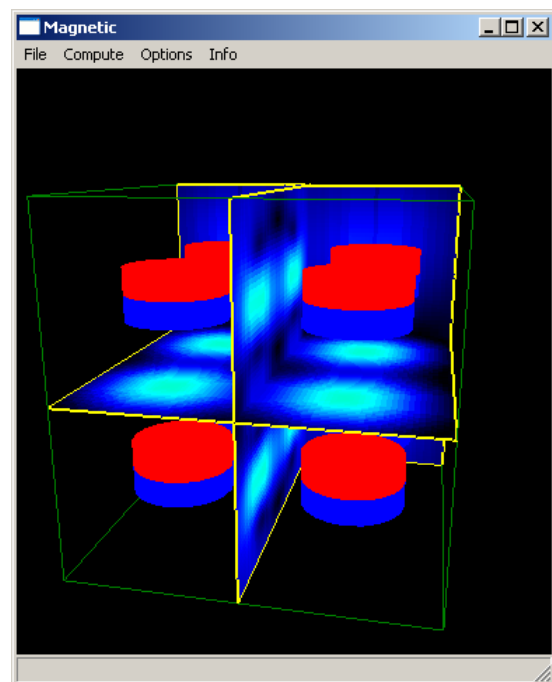


Figure 10: Eight magnets and visualization of their inductions on three planes

The lists of the magnets in the space and the computed field can be saved to a self-developed file format.

Finally, a text file can be given which contains the parameters of the magnets to be used, so after loading them the fields can be computed.

2.5. User-interaction

The user can insert new magnets, delete, move, rotate, resize an existing one, or change the current intensity or remanent induction via user menus.

By clicking the left mouse button above a given magnet it can be selected in the viewer window. In this case the same magnet adjusting dialog window appears which can also be chosen from the menu (Figure 15). The value of the magnetic induction vector can be queried in any point of the space (Figure 16). The graphical objects can be visualized with colours or transparency (Figure 11, Figure 12), thus we can concentrate important details for us.

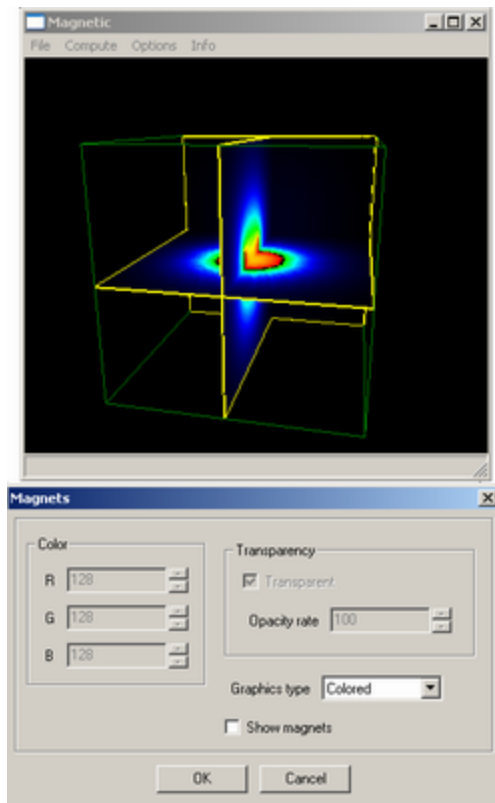


Figure 11: Visualization of a magnetic field by cross-sections (uncheck Show magnets option)

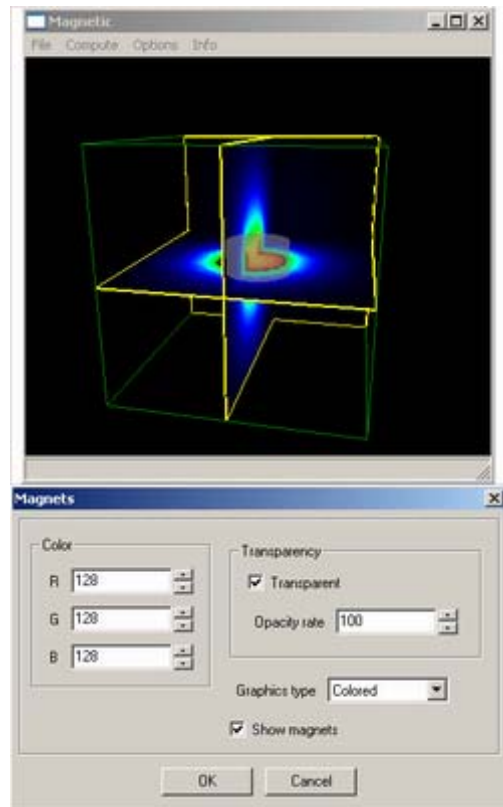


Figure 12: Visualization of a magnetic field by cross-sections (check Show magnets option)

3 Implementation Issues

The application was implemented in C++ (Visual C++ .NET) – because of the flexibility and easy optimization – and based on wxWindows 2.4.2. We used the wxDesigner 2.13c dialog editor utility for developing the GUI, in order to faster running, and the OpenGL for rendering. The UML diagrams of the application are made with SmartDraw 6.1.

The program is optimized to run on an ordinary computer used nowadays, so if available, the program uses the SSE or 3DNOW! processor instruction set. Multiple threads running could be another task for further optimization.

The production environment has the following components.

- Processor: AMD AthlonXP 2000+ (SSE/3DNOW support)
- Graphics card: GeForce4 Ti4200
- Memory: 768 MB

The next table (Table 1) shows the program memory usage and the times necessary for loading the data. Here “Space” shows the dimensions of test spaces in native measure. The measure can be meter or millimetre, but the computation does not depend on it. “Mem” is the required memory. “First load.” is the loading time of single magnet (at first load). “Further load.” is the loading time of any other magnet (after the first load). “Trans” is time of single magnet translation in the space.

Space	Mem	Generation	Further load.	Trans.
20x20x20	8 MB	0.44 s	~0 ms	~0 ms
50x50x50	119 MB	6.6 s	~0 ms	15 ms
70x70x70	242 MB	53 s	15 ms	62 ms

Table 1: Execution times necessary four generation, loading and translation of base spaces

4 Results

The system is suitable to compute and visualize magnetic fields constructed by using the graphic interface. The magnets can be defined by parameters like their geometric sizes. These magnets can be positioned in the available space arbitrarily. In this way arbitrary magnet configurations can be created and simulated. The simulated magnetic field can be visualized in different ways or can be stored for further use in a standard format.

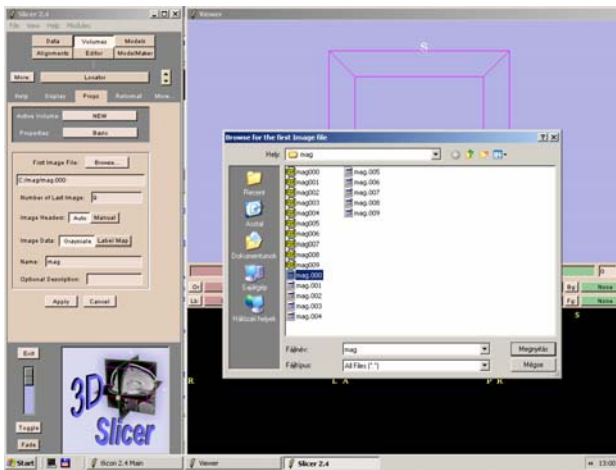


Figure 13: Get input data (slices of the magnetic field) to 3D Slicer

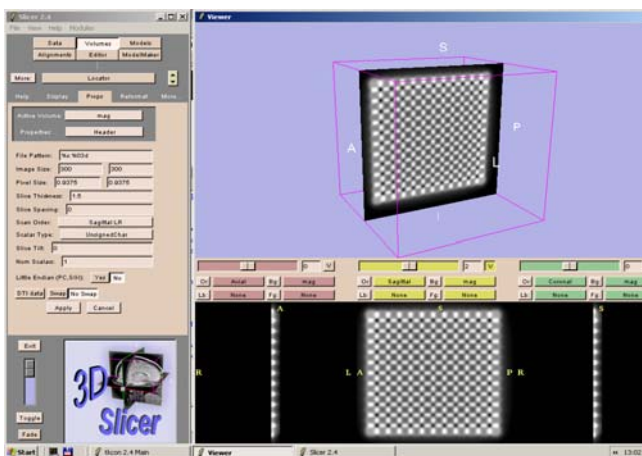


Figure 14: Presentation of the magnetic field by 3D Slicer. Perspective view (above), so-called sagittal, transversal, and frontal slices of the 3D space

The system is able to find some magnet configurations automatically as well. If the user

determines a certain function to be optimized on the magnetic field, then the program can start an optimization procedure trying to find the magnetic field (and equivalently, the corresponding magnet configuration) giving the optimal function value. For example, we can find the configuration giving the highest total magnetic induction gradient value with two, three or more cylindrical magnets. This possibility will be used when a device is to be planned for creating the most suitable magnetic field for medical treatments.

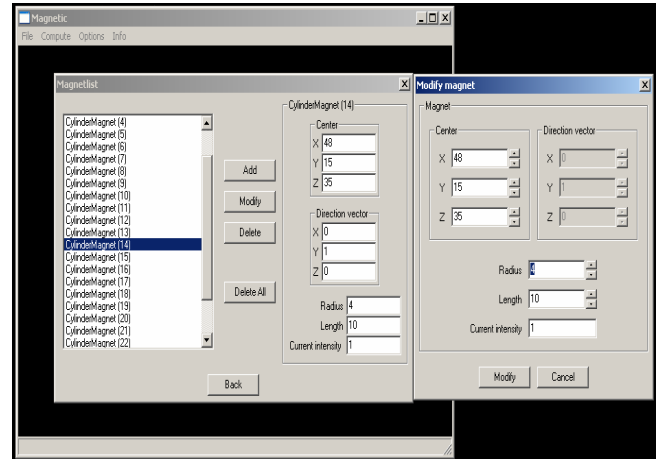


Figure 15: Window for modification of magnet parameters (magnetic program screenshot)

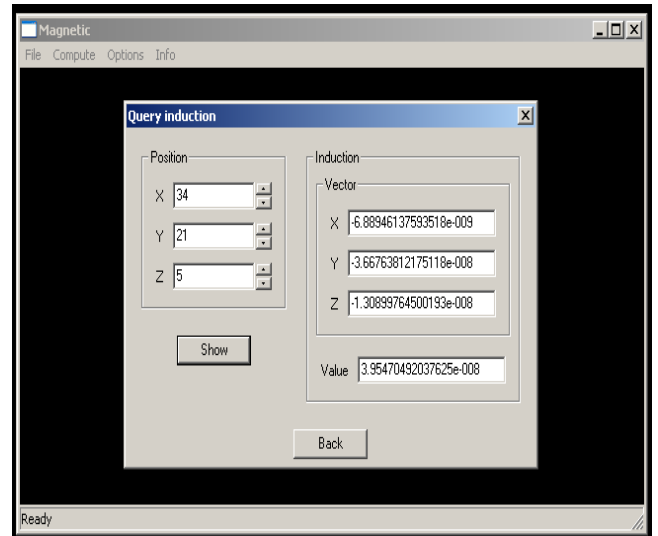


Figure 16: Window for querying the magnetic induction in given point of the space (magnetic program screenshot)

The function of visualization of our system is illustrated by the Figs Figure 10, Figure 11, and Figure 12. As it is shown the user may select among several presentation ways. For example, the fields can be presented as grayscale or color images. These images are cross-sections of the total 3D structure. There are possibilities to give a kind of 3D presentation by Slicer (Figure 13, Figure 14).

Further information about our system can be found at <http://www.inf.u-szeged.hu/~kuba/magnet/magnet.html>.

Acknowledgements

The authors thank Dr. Attila Kuba and Dr. János László for their supervision of this work.

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