Epistemological aspects of the history of painting

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Abstract

The aim of the paper is to study the analogies between the development of painting and the history of geometry. It starts with brief comments on geometrical aspects Renaissance painting, followed by analyses of geometrical aspects of mannerism, baroque, impressionism, and ends with discussion of cubism and abstract art. The paper represents a sequel to our paper *History of Geometry and the development of the Form of its Language*. Its main point is that some painters of the baroque era used the same pictorial form (in sense of Wittgenstein), as Lobachevski and Beltrami in their works on non-Euclidean geometry. We argue, that a similar analogy of pictorial form can be found between impressionist painting and Kleins *Erlanger program*, or between cubism and topology.

Painting and geometry interacted in history in many ways. Perhaps the most extensively studied example of these interactions is the discovery of perspective. Nevertheless, there are many other examples indicating a deep unity of the developments of painting and geometry. What unifies both the art of painting and the science of geometry is the miracle of sight.

A important limitation to the use of the geometrical tools in the interpretation of the historical development of painting lies in the crucial difference in the way in which art and mathematics relate to tradition. In mathematics, every new stage in the development of a theory has to recapture the basic achievements of its predecessor. This feature of the development of mathematics found its expression in that each form of language incorporates the preceding form. Accordingly, in the development of the form of language in geometry, the systematic incorporation of the form of language into language played an important role. This incorporation turned the development of the form of language in geometry into a linear evolution. On the other hand, in painting, each new form appears as an opposition to previous forms. The painter is not interested in recapturing of the basic achievements of his predecessors. Very often, to the contrary, he even consciously avoids them. Accordingly, in the development of painting, the process of the incorporation of the form of language plays a less important role than it had in the development of geometry. Surely, on the purely technical level, the painter would be able to paint in the same way that his predecessors had painted; he mastered their basic technical methods. But in his own painting, he does not rely on these methods in such a strong way, as in geometry, where the new form of language relies on the old one. Therefore the development of the form of language in the art of painting is more fragmentary than in geometry, there is no linear evolution as in the development of geometry.

1. Perspectivistic form of language and Renaissance painting

There is an extensive literature about the geometry of the Renaissance paintings (Berkman 1949, Kline 1953, Pirenne 1970, Edgerton 1975, Blatt 1984, Hagen 1986, Field 1997). The Renaissance paintings are usually analyzed from the point of view of the methods of projection that the painters used. These methods are thoroughly studied and well understood. Nevertheless it is surprising to discover, that these studies are limited to the Renaissance. It seems as if after this period, painters had not contributed anything of importance from the geometrical point of view, as if after Renaissance the roads of art and geometry would have proceeded in separate ways. We believe that this position, which has never been explicitly formulated, but subsists in an implicit form in most texts on the history of art, is incorrect. In our view, the dialogue between painting and geometry continues up to the present day. The only thing that has changed after the Renaissance is that the interaction between geometry and art happens at more deep levels, not just on the explicit, technical surface of painting. Our aim is to follow the hidden, implicit dialogue of painting and geometry beyond the Renaissance. Accordingly we divide the chapter dedicated to the analysis of the Renaissance painting into two parts. In the first part we will present the technical issues of Renaissance painting, which have been discussed many times. This part dedicated to the explicit ***geometry of Renaissance painting*** will be rather brief, and we will soon turn to the more subtle and hidden role of geometry in the ***esthetics of Renaissance painting***.

1.1 Geometry of Renaissance painting: Giotto and Lorenzetti

In most histories of art, the work of the Florentine painter Giotto di Bondone (1266-1337) is considered to be one of the supreme accomplishments of medieval art, and we agree that his use of themes as well as iconography set him well within the medieval period. On the other hand, we can find in his paintings features, which are typical of Renaissance art. These features are related to the geometrical aspect of his paintings, in which a discernible effort to represent real geometrical space and the arrangement of objects in this space can be

seen. Because the subject of our interest is precisely those geometrical aspects of painting in which Giotto was well ahead of his times, we will choose his fresco *Revelation to Fra. Augustin and the Bishop* (around 1325) as our first illustration of the principles of Renaissance art.

As we can see from the auxiliary lines, complemented by F. Kaderavek (Kaderavek 1922), Giotto already understood that in order to evoke the illusion of parallel lines he had to paint lines which converge to a common point. Yet in the fresco before us, the point of convergence of lines belonging to the ceiling and the point of convergence of lines belonging to the canopy are different. It is clear from the painting that there are two main points (points of convergence of lines leading to the depth of space) and also two horizons. That he uses different main points and different horizons means that the perspective accomplished in Giotto's fresco is rather „fragmentary", that is, different parts of the architecture are painted from different points of view.

The next painting with which we would like to illustrate the discovery of the principles of perspective comes from Ambrogio Lorenzetti (1285-1348) and is called *Annunciation* (1344). From the geometrical point of view, there is a very interesting representation of the pavement in the painting. Lorenzetti uses the pavement to evoke the illusion of the depth of space. The lateral sides of the tiles forming the pavement converge to a unique point, which is thus the main point of the painting. This construction of the painting is in accordance with the principles of the perspective. Nevertheless, if we draw into the pavement the diagonals of the tiles, we will discover, that they form a curve. From the geometrical point of view, the perspective is false, as in reality the diagonals form straight lines, and in the central projection, the image of a straight line is always a straight line. Accordingly the line formed by the diagonals of the particular tiles of the pavements should be a straight line.

Thus the question of the correct representation of pavement posed an important problem. We see that Lorenzetti already knows that the tiles of the pavement should gradually diminish, but he fails to surmise the precise ratio of this diminishing. The correct rule of the diminishing of pavements was discovered by Leon Battista Alberti, and we will return to the problem later.

1.2 Esthetics of Renaissance paintings: Ucello and Leonardo

A perfect knowledge of the principles of perspective makes it possible not only to use them to create the illusion of the depth in the picture, but through a slight betrayal of these principles it is possible to intensify the esthetic appeal of the painting. Let us illustrate the idea with the fresco of Paolo Ucello (1397-1475) of *Sir John Hawkwood* (1436).

From the geometrical point of view, this fresco is interesting as the pedestal is represented in a rather strong lower view, while the equestrian statue itself is represented in a direct lateral view. This technique lightens the whole painting and at the same time gives it grandeur. If the whole painting, and not just the pedestal but also the statue, were represented from the lower viewpoint, the statue would dominate the space. Thus by representing only the pedestal from a lower viewpoint, Ucello achieved the impression of grandeur without the negative effect of domination. It is known, that the first version of the fresco was not accepted by the council of the Florentine Duomo and they forced Ucello to destroy the original fresco and paint it anew. Accordingly, we cannot exclude the possibility that, in the original version, Ucello had painted the pedestal and statue from a lower viewpoint. We can only surmise that this gave rise to indignation in the Council. The explanation of the split view as a consequence of Ucello's negligence in the repainting of the fresco is not, in our view, plausible. Ucello was too good a master of perspective to make such mistake. As Giorgio Vasari has written about him: „*This man, endowed by nature with a penetrating and subtle mind, knew no other delight than to investigate certain difficult, nay, impossible problems of perspective. ... Now Paolo was for ever investigating, without a moment's intermission, the most difficult problems of art, insomuch that he reduced to perfection the method of drawing perspectives from the ground-plans of houses and from the profiles of buildings, carried right up to the summits of the cornices and the roofs, by means of intersecting lines, making them foreshortened and diminishing towards the centre, after having first fixed the eye-level either high or low, according to his pleasure. He discovered, likewise, the method of turning the intersections and arches of vaulted roofs; the foreshortening of ceilings by means of the convergence of the beams"* (Vasari 1568, pp. 131-132). It is more probable, thus, that the split view in the fresco was chosen purposefully by the artist, and that the rejection of the first version of the fresco was only an incentive for Ucello to come up with the trick of a split perspective. Of course from the purely geometrical point of view it is a betrayal of the rules of perspectivistic representation. But this betrayal follows clear esthetic goals. It introduces tension into the painting; we find it

impossible to fix unequivocally our vantage point and are forced to equivocate continually between the lower and the lateral views. In this way Ucello decreases the dominance of the painting and magnifies its grace.

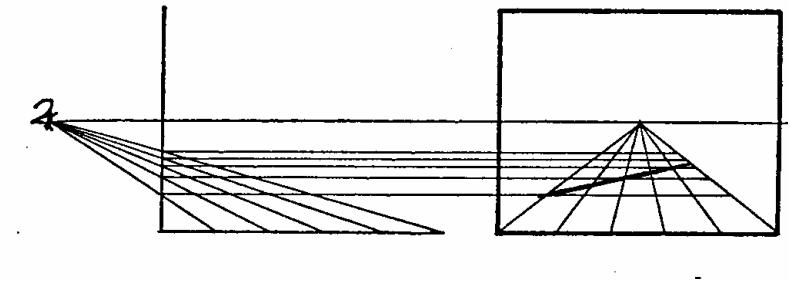
As in many other aspects of art, in the subtle artifice of perspective, Leonardo da Vinci (1452-1519) achieved a supreme mastery. A good illustration of the esthetic use of the principles of perspective is his masterpiece *Mona Lisa* (1505). From the geometrical viewpoint, there is one interesting particularity: „*The horizon on the left side seems to lie much lower than the one on the right. Consequently, when we focus the left side of the picture, the woman looks somehow taller or more erect than if we focus the right side. And her face, too, seems to change with this change of position, because, even here, the two sides do not quite match"* (Gombrich 1949, p. 220). Thus this ambivalence of the position of the line of the horizon in the painting makes it impossible for us to take up an unambiguous view about it. This effect resembles the fresco of Ucello, but it is achieved by more subtle means. The painting evokes a permanent shift of view between a subtle lower field of sight and a direct sight, but the effect is finer than in Ucello's fresco, and so we do not realize it. The broken horizon makes it impossible for our eyes to come to rest at a definite level. It forces us to waiver between the two positions. The uncertainty of our own viewpoint is projected onto the uncertain look of Mona Lisa herself, which gives the picture its famous touch of mystery.

Of course, this is not the only means by which Leonardo achieved the esthetic qualities of his paintings, and maybe even it fails to be one of the most important ones. Nevertheless, from the point of view of geometry it is very important, because the split perspective shows that the mastering of the principles of perspective did not bring the evolution of the geometrical aspects of painting to a rest, but only shifted them from the explicit technical level to more subtle esthetic ones. And so, the study of the use of geometrical means in achieving esthetic goals enables us to follow the parallel between the evolution of geometry and painting also into areas, where direct and immediate contacts are less visible. We have in mind, first of all, baroque art. Geometry is, thus, present in painting not only as a substratum of rules which make it possible to achieve spatial representation, but also in a more subtle way, as rules to be broken to achieve very specific effects in the spectator. The betrayal of the principles of geometry in art can be used to increase the power of a painting.

2. Partitive form of language and the transition from Renaissance to Mannerism

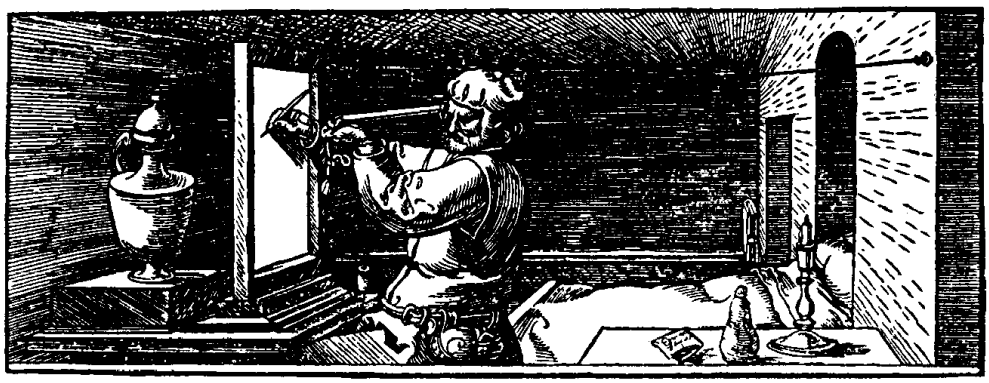
In the analysis of the painting of Lorenzetti we mentioned the problem of the painting of pavements. Lorenzetti used an approximate rule, which, even if not absolutely correct, was good enough to create a rather good illusion of depth. It is probable that, if our attention would not be drawn to the diagonals of the tiles, we would never realize that there is something wrong in the picture. For the construction of more complex pictures, especially in drawings of architecture, it is important to know the correct rule for drawing the pavement, in order to be able to relate correctly the different parts of complex architecture. The theory of the correct drawing of pavement stems from Leon Battista Alberti (1404 - 1472), who explained it in his book *De Pictura* published in 1435. In his explanation of the principles of the perspective, Alberti used an illustration, which from the

geometric point of view, we find interesting.



This picture represents a painting of a pavement. It is interesting, because it represents the painting twice. Once viewed from side, once viewed from front. That means that also the elements of the form of language (horizon, point of view, background) are represented twice. The horizon of the painting viewed from side is reduced onto a single point, but that is not important. What is important is, that we have two horizons in Alberti's illustration. One horizon is a point; the second is a line. Another interesting feature of the Alberti's picture is that the whole representation is flat. Alberti does not want to create the illusion of depth. The picture contains two points of view. Precisely because the painting of the pavement is represented in the picture twice, Alberti was able to derive his rule. The eye of the painter looking from the side on a painting, which is drawn in the left side of the picture, **is the same eye**, as the eye from the point of which the whole painting is seen in the right side of the picture. Both times it is the same painting. In the left side of the picture Alberti shows how the eye sees particular tiles of the pavement. But because it is the same eye as that from the point of which the painting in the right side of the picture is seen, the sizes of the tiles must be, from both viewpoints, the same. Accordingly, Alberti represented how the eye of a painter sees the tiles first in a cross section, and then he transferred the sizes of the tiles into the frontal view. This technique lies at the core of Alberti's method.

Thus, we can say that in Alberti's picture there are two internal points of view. (Internal means belonging to that what is represented in the picture. An external point of view belongs to the way how the external spectator see the picture.) The picture of Alberti is flat, so it does not make use of an external point of view, and both of the points of view belong to the painter whom is Alberti representing. We believe that ***the idea to connect two pictures of the same object into one figure is the central idea of the whole of descriptive geometry***. Consequently, we are able to say that Alberti's means of representation of the projection of the pavement was the first step towards descriptive geometry. Monge's projection is based on a similar duplication of the picture of a particular object.



Let us now turn to the print of Albrecht Dürer (1471 - 1528) found in his book *Underweysung der messung mit dem zirckel vn richt scheyt* printed in the year 1525. We are able to see that, in this picture, the image contains a duplication of the point of view. But this time one point of view is internal (it is the eye of the painter) while the other is external (it is the point of view, from which the perspective of the whole image is constructed). As we have shown (Kvasz 1998, p. 143-146), this picture of Dürer represents the configuration, on which the whole of projective geometry is based.

We would like to introduce one more painting, produced by Hans Holbein (1498 - 1543) in 1533, called *The Ambassadors*. We find this painting interesting because it also contains a duplication of points of view. In this case, both viewpoints are external. That both of the points of view are external means they are related to the spectator who looks at the painting. From one point of view, which is directly in front of the painting, we see two ambassadors clothed in contrasting attires, standing in a room with sumptuous surroundings. From the frontal view, we see by the feet of these ambassadors the hazy image of what could be described as „a strange splash." In order to see what „this splash" represents, we have to take up a different point of view, located at the left side of the painting, close to the frame. (Actually, the frame of the painting contains a

small hollow, indicating the correct place where one is supposed place the eye to see the image.) If we look at the painting from this extremely acute angle, „the splash“ turns out to be a perfect skull (Leeman 1975, pp. 18). Holbein's painting is based on the epistemic subject, understood as the viewpoint from which we look at the world. Let us interpret this painting by placing it alongside the pictures of Alberti and Dürer. That we place these images side by side means that we interpret Holbein's innovation as parallel to the duplication of points of view made by Alberti and Dürer. While in Alberti, both points of view are internal; and in Dürer, one is internal and the other is external; we find that in Holbein both are external. Accordingly, we suggest that the form of language of Holbein's painting, placed beside Alberti's and Dürer's, completes them by adding a third possibility, which the duplication of points of view opens to us. Thus, the three pictures mentioned above represent three possibilities open to us in the development of the form of language of perspective. We already used Dürer, in our paper on history of geometry, to establish the basis for the emergence of projective geometry. We could use Alberti's picture, in a similar way, to underscore the ideas from which descriptive geometry emerged. We are concerned here not with the technical details of the drawing, but rather with the basic epistemological nature of the image. This image contains two well-coordinated representations of a particular object. Even if the two representations, viewed in isolation, capture only some of the aspects of the depicted object, we note that by coordinating both views, we are able to reconstruct the complete information about the represented object. To develop representations, which contain the complete information about represented objects, is the goal of descriptive geometry.

2.1 Esthetics of Mannerist painting: El Greco

In the previous section we discussed the geometrical aspects of the transition from Renaissance to Mannerism. Now we would like to turn to some of the esthetic aspects of Mannerism. We will concentrate on those esthetic aspects, which are based on the breaking of the rules of geometry. For this purpose we have chosen one painting of Domenico Theotokopulos (1541-1614), better known as El Greco. The elongated shape of his figures is a ***geometric*** deformation, and so it presents a challenge for us to offer its geometric explanation. Of course, geometry is only an auxiliary tool for painting, and so what we will offer cannot be more than an elucidation of a single aspect of El Greco's art. We believe, that an attempt to give a geometric explanation of the elongated shape of the figures in some of his paintings discloses another dimension of his artistic creativity. It is very probable, that the effect, which we want to describe here, he was totally unaware of. El Greco had an artistic aim and the painting, which we are going to analyze, is first of all a beautiful and magnificent work of art. We believe, that due to our method of analysis we will be able to disclose some new aspects of his painting.

Let us now turn to the painting *John the Baptist* (around 1600). Let us stand in front of the painting at a maximal distance, which the hall of the gallery will allow us, direct our view on its chest, but without fixing it on any particular detail of the painting, and let us walk slowly towards the painting. The effect, which we can observe is that the figure of John the Baptist seems to leave the painting and levitate in front of its surface. This rather remarkable effect can be empirically tested on many other paintings of El Greco.

A possible explanation of this effect is following one: From experience we have learned to estimate our distance from a person out of the magnitude of its appearance. We know how tall an average person is and so when we look at a man from a distance, we know approximately how far away he is from us. Now when we move and the person is not too far away from us, his picture on our retina changes. From experience we have learned to transform the changes of the projections of a person on our retina into the feeling of the change of his distance. Thus instead of the impression, that the person is growing larger (what is exactly what happens with the projections on our retina), we feel, that he is approaching us. And this is precisely the moment, in which the elongation of El Greco's figures starts to be relevant. When we slowly approach the painting *John the Baptist* with our view not fixed on any detail, the described mechanism is all the time transforming the enlargements of the projection of John the Baptist into our retina onto the feeling, that the painting is still nearer. But El Greco succeeded to fool this mechanism and he achieved, that the elongation of the figure is transformed into the feeling of its greater nearness. The figure of John the Baptist is elongated only slightly, and so when we look at the painting without fixing our view on any particular detail, we do not perceive it as an elongated figure any more. Our motion destabilizes the mechanism, which in an automatic manner transforms the height of a person into the feeling of his distance from us. Due to this destabilization the mechanism transforms the additional height of the figure (its elongation) into the feeling of its greater nearness to us and thus the figure leaves the surface of the painting. The reader can easily test this effect himself in any gallery owning El Greco's paintings.

The effect of levitation of the figure of John the Baptist in front of the paintings in accordance with our interpretation of Mannerism as a period, based on the split of the subject. This effect is based on a conflict between two subjective mechanisms, the one which transforms the height of a person into the feeling of its nearness and the other which compensates the changes of visual perceptions during our motion. Thus El Greco used a distortion of these subjective mechanisms in a similar way as Leonardo used the distortion of the horizon in order to evoke the mysterious smile of his *Mona Lisa*. What remains is the explanation of the meaning of this effect in the context of El Greco's artistic aims. El Greco was not a mathematician, but an artist, an such geometric tricks, if he would have ever paid them any attention, were from the point of view of his creative intentions surely marginal.

It is important to realize, that El Greco was raised in the Byzantine tradition of painting, to which, among others belonged also Andrej Rublov, who painted similarly elongated figures. When El Greco came to Venice he brought all the elements, which in the Byzantine style were rather natural, into a totally new context of post-Renaissance painting (Kelemen 1961). He placed the Byzantine figures into a Renaissance space. In this way the figures were liberated from the flat surface of the icons and started to ascend into space. Thus in the levitation of John the Baptist in front of the surface of the painting El Greco seems only to be repeating his own personal experience from the time, when he came to Venice and saw the Byzantine figures liberated from the flat surface of the paintings for the first time. El Greco experienced something, what maybe no other artist of the

western tradition could ever experience. It is because in the western tradition the perspective was discovered step by step in the span of several generations, and so everyone could experience only a small shift. El Greco, however, learned to paint at Crete in the Byzantine tradition, and as a young man he left his home and came to Venice, where he learned the perspective. That means, that with full understanding of the technical aspects of painting and with fully developed ability to reflect and conceptualize his feelings he experienced the opening of the pictorial space. Of course each of us has made the same experience, but in our case it happened in our childhood, when we were not able to understand and conceptualize this experience. El Greco, however, has made that experience as an adult man and as a painter, and therefore he was prepared to fully understand what he experienced and it is this experience of the opening of the pictorial space, which is expressed in his paintings.

3. Interpretative form of language and baroque

The aim of this chapter is to show that the interpretative form of language, with the help of which Lobachevski, Bolyai and Gauss discovered Non-Euclidean geometry, has a parallel in baroque painting. The thesis we develop is at variance with the way the connection between painting and non-Euclidean geometry is usually interpreted. Historians of art usually make the connection between non-Euclidean geometry and cubism or with early stages of abstract painting (see Henderson 1983, or Adelman and Compton 1980). They base this interpretation on the views expressed directly by the painters themselves, painters such as Braque, Malevich, Kandinskij, or Duchamp in various manifestos of cubism, futurism, or suprematism. They find support for this interpretation in contemporary theoretical reflections, in which we find the direct mention of non-Euclidean geometry. Nevertheless, we believe that the parallels made by this interpretation, even if were expressed by the painters themselves, are misleading. They are due to a historical accident: at the time of its discovery (in the decade of the 1820's), non-Euclidean geometry was rejected and totally ignored by the scientific community as well as by the public. Only after the death of Gauss in 1855, when his diaries became available, did the mathematical community discover that the *Princeps Mathematicorum* (or uncrowned prince of mathematicians, as Gauss was sometimes called) took the possibility of non-Euclidean geometry absolutely seriously. Even if he never expressed his conviction publicly, in his private correspondence and diaries it became clear that he had harbored no doubts with regard to the existence of the non-Euclidean geometry. The term 'non-Euclidean geometry' itself is Gauss' coinage. Gauss' authority led to a reconsidering of the general attitude towards the non-Euclidean geometry and, by connection, to a rediscovery of the forgotten works of Lobachevski and Bolyai.

Accordingly, non-Euclidean geometry started to permeate the awareness of the general public in the 1880's. Its full acceptance only came during the first decades of the 20th century, due to its relationship with the theory of relativity. Thence, non-Euclidean geometry drew vivid discussions and the views of the painters we mentioned were formed in the atmosphere of these heated discussions. Non-Euclidean geometry, cubism and futurism were connected in the minds of people basically due to a lag in time, by a half-century, in the acceptance of the non-Euclidean geometry. These painters were fascinated to hear about the possibility of the new geometry. We can understand their enthusiasm for the new geometry. They saw their own efforts in overcoming the representational tradition of classical painting as parallel to the efforts of the founders of the non-Euclidean geometry, who overcame Euclidean traditions in geometry. We believe the new geometry would have been greeted by painters with the same enthusiasm a half-century earlier. Joseph Turner and William Blake would have, no doubt, taken up non-Euclidean geometry, had its permeation into the awareness of the general public not been retarded as a result of Gauss' caution and the nescience of the scientific community. In every age there are painters who look for new means of artistic expression. They will feel allied with mathematicians, who do something similar in their field. Nevertheless, such an alliance remains an ***external parallel***, based on the parallel between the social and psychological situations of the painter and the mathematician. It has little to do with the content of their work.

3.1 Geometry of baroque painting

As we have already mentioned, the basic thesis that we want to justify is that, from the epistemological point of view, baroque painting represents a transition to the interpretative form of language. Thus, baroque painting has features, which are parallel to non-Euclidean geometry. We will base our interpretation on the analysis of paintings of major figures of the baroque era. The first painting that we would like to analyze is *Las Meninas* of Diego Velázquez (1599-1660). This masterpiece was painted in 1656 and can be seen in Prado Museum in Madrid. The painting has been interpreted many times. We will not enter into the full complexity of this painting, but we will restrict our focus to the geometrical approach that was explained above. We believe that by thus restricting ourselves to geometric tools of interpretation we may say something new about this painting.

This painting depicts the painter working in his studio. First, we are unsure of what he is precisely painting. Only after some time we discover (or perhaps find it in the literature) that at the far end of the studio

there is a mirror, in which we can see the royal couple sitting as models. At the first sight, it could seem that this painting is parallel to Dürer's drawing, which also represented the painter at work. Both paintings are painted from an external point of view and represent the process of painting. Thus, it seems that the painting of Velázquez could be based on the projective form of language. But after some reflection we can find out that this is not the case at all. In Dürer's drawing, the other point of view is internal. The drawing represents the process of projection, which proceeds from the model through the transparent foil and into the eye of the painter. On the other hand, Velázquez's painting fails to represent the way the painter sees his model. Thanks to the mirror we know what the painter is looking at, but strictly speaking, his line of vision is not represented in the painting. For the external point of view (from which we are looking onto the whole painting) the figures of both Monarchs are hidden. Accordingly in Velázquez's painting we cannot follow the process of projection, which connects the model, the painting and the eye of the painter, as we could in Dürer's drawing. The painting resembles the drawing of Dürer, in which the model fell out of the visual field. Every painting represents directly only that what is before the spectator. What is behind his back, cannot be represented directly. It is precisely the royal couple, which has fallen out of the visual field of the external point of view and, accordingly, cannot be represented directly in the painting. Velázquez brings them back into the painting with the help of the mirror.

That means that the mirror represents a ***fragment*** of the world, which the painting cannot represent directly, embedded into the painting. But it is important to realize that the mirror is unconnected to the point of view of the painter. The painter fails to see the mirror, which lies behind his back. The image in the mirror, even if it represents the royal couple, onto which the painter is looking, is not a part of the visual field of the painter; it is not related to his point of view. The painter does not see the mirror, and therefore the image in the mirror is not represented from the point of view of the painter, that is, the internal point of view. The mirror is there for us; we see it from the external point of view, from which we look onto the whole painting. In contrast to Dürer's drawing where the eye of the painter is the internal point of view, in the Velázquez's painting, the eye of the painter fails to be important. Accordingly, it does not belong to the form of the language. It does not organize the image of the painting into a whole, as in Dürer's drawing. If the painter were a servant with a carafe, serving the royal couple, the painting would not lose its meaning. Instead, we would be viewing a short break during which the King and Queen relax. Even if were to replace the painter with a servant, we would still be able to grasp the meaning of the painting. On the other hand, if we could similarly replace Dürer's painter with a servant in his drawing, this would definitely defeat its meaning. The destruction of the internal point of view, which connects the model with its image, does not affect Velázquez's painting, but destroys the meaning of the Dürer's drawing. This means that their form of language cannot be the same. Accordingly, our attempt to interpret Velázquez's painting with the help of the Dürer's drawing was ill-advised. Velázquez's painting does not use the internal point of view. It is not the eye of the painter that unites the image of the painting.

The points of view from which we see the whole studio, as well as that from which we see the royal couple in the mirror, are both external points of view. Thus, it could seem that Velázquez's painting is parallel to the painting of Holbein. However, after further reflection, we recognize that this interpretation will not work either. The problem is that in Holbein's painting, the points from which we are able to view the ambassadors and the scull are different. We have to go with our eye close to the edge of the painting in order to see the scull in the proper perspective. In contrast to Holbein's painting, in Velázquez's painting, the point of view from which we peer at the whole studio and the point of view from which we see the mirror are the same point. This means, that Velázquez, similarly to Beltrami we might add, ***identifies these two points of view***.

We can understand this identification of points of view more clearly if we imagine the following variation in *Las Meninas*. We replace the mirror hanging in the back wall of the studio with a painting. In this painting, the royal couple poses in exactly the same way as they did in the mirror. Everything we see in the mirror, we will represent in this painting. In other words, we transpose the whole of the image represented in the mirror, into the painting. Now, if the painting is precisely the same, from the external point of view nothing has changed. Yet it loses its epistemological meaning. No longer are we able to determine what the painter represented in *Las Meninas* actually is viewing, let alone painting. The painting hung on the back wall of the studio, even if it would contain the same information represented in the mirror, tells us less. It is no longer possible for us to find out what is being painted by the painter depicted in *Las Meninas*. Accordingly, we see that we really have two different points of view here, located in the same point of the space. One is external; it constitutes the general view of the studio. The other is internal; it is connected with the mirror. Thus, we see that we have ***two languages present in the painting***. If we transport the information from the internal to the external languages, from the mirror to the painting, *Las Meninas* loses its meaning. Both Velázquez's painting and Beltrami's model are based on the coexistence of two different languages in them.

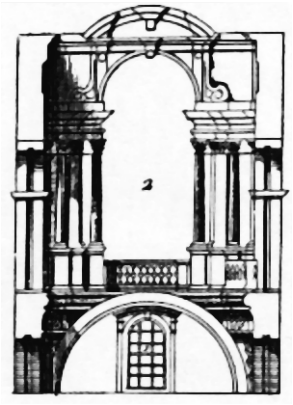
In order to understand the whole meaning of *Las Meninas* it is not enough to see what is represented in the mirror (as the painting could represent the same thing), but it is also important to know that it is a mirror. This knowledge is important because it enables us to understand that the image, reflected in the mirror, represents figures outside of our visual field. We interpret the information in the mirror as a representation of figures which are situated in front of the painting. This act of interpretation is, epistemologically speaking, a ***translation***. Accordingly, the translation that the mirror offers us in Velázquez's painting parallels a similar translation in Beltrami's model. The mirror on the back wall of the studio stands for the circle in the Beltrami model. Inside his circle, Beltrami represents the non-Euclidean world, which **lies outside** of the realm of Euclidean pictures. Similarly in his mirror, Velázquez represents a fragment of the world, which falls out from the visual field.

Even if Velázquez fails to represent non-Euclidean geometry in his paintings, he applies many of the principles, which Beltrami used in constructing a non-Euclidean world. Both Beltrami and Velázquez make use a ***fragment*** (circle, mirror), which is ***embedded*** into the external picture (in Beltrami, Euclidean plane; in Velázquez, the painting of the studio). In both cases, there is a physical ***identification of two points of view*** (the center of projection and interpretative distance in Beltrami; the external view of the studio and the internal understanding of the principles of mirrors.) In both cases, the information lost in physically identifying the two points of view is made up by introducing a ***translation*****.** Beltrami translates the relations from the language of the Euclidean geometry of a circle into the language of the geometry of the non-Euclidean plane. Velázquez translates the picture from the external view of an image on the back wall of his studio into the internal view of the mirror (our imaginary replacement of the mirror with a painting representing exactly the same information removed the possibility of making this translation). Finally, both Beltrami and Velázquez use the new form of language in order to represent something that it is ***not possible to represent directly***. The royal couple lies behind our back and have fallen out from our visual field; non-Euclidean figures lie „behind Euclid's back", and therefore fall outside of any Euclidean visual field.

The second painting, which we use to illustrate the parallels between the form of language of non-Euclidean geometry and of baroque painting is a fresco called *The Apotheosis of St. Ignatius*. It was completed in 1694 by the Jesuit painter, architect, and theoretician of perspective, Andrea Pozzo (1642-1709), and covers the ceiling of the church of St. Ignazio in Rome. This fresco is one of the masterpieces of the illusionist baroque. St. Ignazio is one of the most important churches of the Jesuit Order. It was built between 1626 and 1650. Originally, it was projected to have a high cupola. But neighboring Dominicans were afraid that a high cupola would deprive their library of light. In 1684 the Jesuits and the Dominicans reached a settlement, according to which the church would not have a real cupola, but an illusionist one, painted on the ceiling of the church. Andrea Pozzo, who already had experience in this field, was commissioned for the realization of this project.

In his book *Perspectivae Pictorum atque Architectorum*, which was published in 1693 in Rome, Pozzo explained in detail the method, with the help of which he constructed the ceiling of the church of St. Ignazio and created its illusionist cupola. In Pozzo's book, we find the following illustration.The lower third of the drawing represents the semicircular vault of the church, which really exists. The upper two thirds of the drawing (marked by number 2) represent the fictive architecture of a non-existent cupola. Pozzo made a detailed project of this non-existent piece of architecture and designed its decoration. Until now the illustration does not differ from many other drawings used by architects in projects for rebuilding existing buildings. In such projects, a new part, which the architect is designing, is added on to the existing architecture.

In Pozzo's drawing, the situation is quite different however. He knows that the cupola will never be build. His is not drawing a plan for a contractor, but preparing detailed instructions for a painter. Instead of giving the plan to the contractor, he projects the whole fictive architecture of the cupola onto the vault of the church. In this respect we can see in Pozzo's drawing a parallel to Desargues, because it represents a picture of a picture. Nevertheless we believe that Pozzo goes further than Desargues. Rather, Pozzo's painting parallels the picture with the help of which Lobachevski derived the trigonometric formulas of non-Euclidean geometry.



The similarity between Pozzo and Lobachevski consists, first of all, in the use of a common pattern of reference. Even if Desargues draws a picture of a picture, in the end, upon following the sequence of the projections, we succeed in reaching a real referent object, which is represented in the picture. In Desargues, even

if the reference of the language is mediated by a sequence of intermediate stages, it always ends up in reality. On the other hand, in Lobachevski, and similarly in Pozzo, we never reach reality in the end. The non-Euclidean world and the cupola of St. Ignazio simply do not exist. More precisely, they exist only in our imaginations. (Of course, you could say that also a Euclidean world exists only in our imagination, but that is the next step, taken by Cayley, which we will not consider now). This means, that in Pozzo and Lobachevski, there is an ***encounter of two worlds***: the real and the fictive. And in both cases, the two worlds are connected with the help of a ***projection***. Lobachevski projects a Euclidean triangle from the limit sphere into a non-Euclidean plane, while Pozzo projects the non-existing cupola onto the existing vault. With the aid of a projection, Lobachevski ***transfers*** trigonometric formulas from the Euclidean limit sphere onto a non-Euclidean plane; Pozzo transfers the metric relations among different parts of architecture from the non-existent cupola onto the existing vault. If we realize that the trigonometric formulas as well as the relations among parts of the architecture determine the metric structure of the space, we can say, that Lobachevski and Pozzo transfer the metric structure of one world unto another.

In the resulting mural, Pozzo achieves a continuous transition from the architecture of the walls on the sides of the church into the architecture of the illusionist cupola. He decorated the whole cupola with a number of figures, sitting on different ledges, portals, arcades, or just hovering in space. With these figures, we are confronted with a double situatedness, which is typical to the interpretative form of language. In Lobachevski, every line has a double-meaning: on the one hand, it is just an ordinary Euclidean line drawn on Euclidean paper; on the other hand, we are to interpret the line as the side of a non-Euclidean triangle. A similar double-situatedness is present in connection with the Pozzo's figures: from the external point of view (the point of view which refuses to accept the illusions and to enter into the imaginary world of the non-existing cupola) these are just some figures painted on the ceiling of the church. Yet, from the internal point of view (the point of view that accepts the game and enters into the world of illusions), these are figures sitting on the arcades of the cupola of the church, towering into the sky. Like in Lobachevski, in Pozzo each figure is situated simultaneously on two backgrounds. One of them is the background of the really existing ceiling, which is a parallel to the Euclidean plane, on which Lobachevski draws his pictures; the other, the background of the fictive cupola, an analogy to Lobachevski's non-Euclidean plane.

We may raise the following objection: every painting that represents something can be interpreted in two ways. Either we may refer to the elements, which are actually present on the canvas, or to the reality that these elements represent. We can always reject the game, which the picture invites us to play. We can reject the illusion of depth in perspectivistic paintings and stubbornly insist that what we see in the painting are not the parallel sides of a ceiling, but only two convergent lines, which intersect in the middle of the picture. Nevertheless, the two ways we have of interpreting a picture, which we confront in Pozzo and in the non-Euclidean geometry, are subtler, as we argue below. We are not confronted with the crude alternative of accepting or rejecting a pictorial game. The alternative, as we see it, is not between entering the world of the picture or staying outside of it. In geometry, we could equivalently suggest that there are no real Euclidean lines at all on a sheet of paper. Instead, we could say that these lines are of stains of ink of rather complicated form splattered across the sheet of paper. And if we would look more carefully (e.g. through a magnifying glass), we would have to accept, that two lines do not meet at a single point, but rather in a small quadrilateral region of varying thickness, having rather complicated boundaries.

When we refer to two ways of interpreting the figures drawn in Pozzo's painting, we are not confronted with a radical opposition. In Pozzo, both alternatives remain inside the world of the painting. The painting itself, without denying its pictorial nature, can be read in two ways. It is similar to Lobachevski's figure, where both the Euclidean as well as non-Euclidean readings remain within geometry. We can understand the subtle difference more clearly, if we walk along the church, away from the point at which the illusion of the cupola is perfect. If we walk in the direction of the altar, the illusion of the cupola gradually disintegrates. Despite this, the ceiling remains, from the view of the spectator, nicely decorated. The structure of the inner language can be destroyed without the painting ceasing to be a painting. The figures still hovering in space above our heads, they are still situated among lines of architecture, which convey depth. The painting still preserves a local persuasiveness, but the global illusion of a cupola gradually vanishes.

Pozzo aims at something more than simply to create the depth of space. He uses this space in order to situate, within this space, a cupola. The cupola is the intended background for the decorative figures. When the illusion of the cupola falls apart, the space in which it was created still remains, and so the figures designed to decorate the cupola are still situated in this space. Only one layer of the meaning of the painting has vanished; but not the pictorial nature of the lines as such. It is this double situatedness inside the language itself, which connects Pozzo to Lobachevski. As we explained in our paper (Kvasz, 1998), this double situatedness is more clearly present in Beltrami, who identifies an internal point of view with an external point of view. The internal viewpoint is the center of projection. (Lobachevski projected, with the help of this internal point of view, a limit surface onto a non-Euclidean plane.) The external viewpoint created the interpretative distance. (Lobachevski

bridged, with the help of such an external viewpoint, the precipice separating the Euclidean and the non-Euclidean worlds.) We express the identification of the external and internal points of view, in our paper as follows: „*The central problem of Lobachevski was that in his pictures he had a conflict of grammars. Beltrami removed this conflict when he identified the implicite interpretative subject with the explicit projective subject. In other words, he looked at the picture of Lobachevski from the improper center of projection"* (Kvasz 1998, p. 151). Thus, Beltrami transposed the external viewpoint, from which the picture has to be interpreted, into the internal point of view from which Lobachevski originally constructed it. Pozzo makes a similar identification. He identifies the external point of view, that is, the point from which we have to look onto his fresco in order to see the cupola, with the internal one, that is the center of projection from which he constructed the cupola. This cupola is painted on the flat ceiling of the church, which we view from a double point of view. This identification strikes the eye when we move along the church, away from the central point from which we ought to look at the ceiling (in illusionist architecture, this point is marked usually with a cross, which is engraved in the pavement or represented by tiles of a different color). As we move away from this special point, the illusion of the cupola begins to dissolve, and we are able to discriminate the two backgrounds.

The shattered illusion shows that the interpretative form of language makes use of the splitting of the subject, which we saw in the projective form. The interpretative form of language identifies both viewpoints, moving them into the same point. Thus, we are able to achieve an interpretative distance from ourselves. The projective form (for instance in Dürer's drawing) also creates a kind of distance. It creates this distance by situating one part of the split subject directly into the picture, in the form of the internal point of view. It then uses the other part of the subject to examine what the first one is actually doing, during the process of painting. In the interpretative form, interpretative distance is created in another way. It fails to take the form of the external perspective; it fails to achieve a view on us from outside. The interpretative form identifies the external and the internal viewpoints. This identification, this compressing of external distance into one point, is what opens up new, illusory worlds in both painting and geometry. The subject no longer views at itself from an external point in order to understand itself. The subject remains in itself; the interpretative distance is created instead with the help of an interpretative shift. From a single point, the subject creates a double view of the world; two points of view, from which the subject is able to look at the world, lie in one geometric point. By taking two points of view on the world, it is possible to make an interpretative shift of meaning. The shift does not relate to the point from which I am observing the world, but only changes what I see. That we are able to have a ***simultaneous double vision***, to acquire an external and an internal vision from a same vantage point, the actual and illusionist visions, is the innovation that baroque painting brought about. This innovation makes baroque parallel to the non-Euclidean geometry. Accordingly, the epistemic subject is ***doubled*** in baroque painting, and not split, as was the case in Alberti's, Dürer's, and Holbein's artworks.

What we describe is a kind of newly achieved unity, heightened by the experience of the split. Or, has the split been replaced by a doubling? Whatever way we look at it, the discovery of the doubled subject creates a doubled world perception. Beside the objective external world, we obtain an illusionist internal world, be that the world of the non-existent cupola or of non-Euclidean geometry. In painting, this effort to overcome of the split of the subject manifests itself in the attempt to ***pull the spectator into the internal world of the painting***. This aspect relates the baroque to non-Euclidean geometry. Lobachevski was well aware that it is impossible to draw non-Euclidean pictures. But if we forget for a moment about this impossibility, if we submit to the illusion of his pictures, a radically new world opens up before our mind's eye. And this interpretive shift is achieved not by some kind of an external view or some external distance. On the contrary, we have to give up any external view, give up any critical distance, any reservations of reason. Reason knows that what Lobachevski is doing is insane (and many of his contemporaries did not hesitate to remind him of it). We have to give up any reservations of reason and submit totally to the illusions of the figures. And it is precisely the double presence of the subject, in one point of space from which we observe the world, which makes it possible to create a consistent world of illusion. We put the whole burden of the objective external world on the shoulders of the one of two points of view. The two points of view form the doubled subject of the interpretative form of language. In this way, the second point of view, which is located at the same point in space, is freed from the burden of reality and, thus, can enter the world of illusions. When the subject was split (and not doubled), each of its two parts was looking onto the world from a different point of space, and therefore each view carried its own piece of reality. By the identification of the two poles of the split subject we can free one of them from the burden of reality, because it makes no sense twice to carry the same information. This opens up the entrance into a new world, opens up the possibility of an interpretation.

3.2 Esthetics of baroque painting

In Renaissance painting, the painter creates a world on his canvas and the spectator is able to observe this world. The spectator becomes a witness of the scene represented by the painting. Even if the painting engages his attention or captivates him, he still preserves some kind of reservation, some kind of distance from

which he observes the painting and makes his own assessment. By representing the world from the point of view of a spectator, perspectivistic paintings that open a view on a landscape stretching to the horizon, create a certain distance from the scene represented in the painting. If the perspective of a scene opens ***there*** at a specific distance before me, then simultaneously I am ***here*** at the same distance in front of it. Every there has its corresponding here. It is this distance that Baroque painters want to break, the same distance, which gave Renaissance paintings their specific balance and peace. These paintings evoked the undisturbed harmony of figures with congenial surroundings, yet never engaged the spectator. They failed to have a strong, dramatic appeal on the spectator, which Baroque painting endeavored to achieve. We see that Baroque painters strive to break this kind of distance. A baroque painting attempts to pull the spectator directly into the middle of the scene depicted. The painter can achieve this effect by using many different means, only a few of which relate directly to geometry. On the other hand, the very aim to break the distance created by perspective is a purely geometric issue. To go against perspectivistic distance means to go against geometry, or at least to go towards a geometry different from that which is disclosed by Renaissance perspective.

The painter can break the distance and create a perception of closeness simply through his thematic choice. For instance, an intimate, slightly erotic scene may create a feeling of almost physical presence. The opposite extreme in thematic choice may be the dramatic scene, capable of strongly captivating the onlooker. Faced with such a dramatic theme, we forget that we are spectators, and feel that we are participants in the events portrayed. Obviously, the choice of theme in painting has no parallels in geometry. However, besides thematic choice, there are other possibilities, some of which are more or less geometrical, which the baroque painter may employ to disturb the calm distance of Renaissance perspective. The baroque painter may use many or all of these technical possibilities in the same painting. The paintings below illustrate various geometrical principles of the destruction of perspective. They illustrate the particular features we discuss, or may, at the same time, illustrate other principles.

As a first example we introduce the painting *David with the Head of Goliath* (1607), produced by Caravaggio (1573 - 1610). The painting is distinguished by a ***dark, nearly black background***, from which the figure of David seems to spring, we could almost say detach. In the Renaissance paintings the figures were usually situated in a landscape, which stretched towards the horizon, or in a room, which defined the space occupied by the figure. The eye of the spectator could freely follow the paths that mark the depths of space. The space that opened up before the gaze of the spectator, led his sight into the space, and simultaneously, in a kind of countermovement, also defined his position with respect to the space. As the landscape in the background stretches into the distance, this distance constitutes also our position with respect to the landscape, and so creates a distance also from everything placed in this landscape. By hiding the whole background into darkness, Caravaggio destroys precisely the spontaneous creation of distance, based on an automatic identification of the position of the spectator with respect to the background of the painting. Caravaggio leaves us no possibility to determine our position with respect to the space in the painting. The painting has no depth, into which we could push the figure of David. David is not somewhere out there in the country; he is here, directly in front of us.

Another principle, which baroque painters used to break the depth of space in their paintings, was the use of ***lines in oblique directions***. If we look on the *David with the Head of Goliath*, we see that the dominant lines of the painting are drawn in oblique directions. In contrast, the paintings of the Renaissance period usually maintain a horizontal-vertical structure. In such a structure, lines that create depth lead our vision to the horizon. In this manner, they help create a perspectivistic distance. On the other hand, the oblique lines followed by baroque paintings catch our attention and so detract our view from following the direction that leads to the horizon. They hinder our perception of the depth of space. In this way, the perception of depth is systematically

disturbed, even if not to such a degree as the dark background. Using oblique lines in painting detracts our mind from the creation of the perspectivistic distance. As a result, the painting pulls us into the scene portrayed.

A further principle, which the baroque painters often use to disturb our perception of the depth of the space, is the use of different **fabrics**. A good illustration of this principle can be found in the painting *Infanta Margarita* (1653) from the hand of the master Diego Velázquez. *Infanta Margarita* is placed in front of a ruffled curtain. She is standing on a carpet, decorated with a rich pattern. The table on her right side is also covered with a textured fabric. So the eye lacks something to detain the vision on. The spectator is at a loss to locate a particular geometric line, which would help him form a specific idea about the depth of the space, in which the Infanta is placed. In Velázquez, the space fails to be totally missing, as when Caravaggio set David against a dark background. Velázquez fails to destroy the space with a system of diagonals, as does Rembrandt in portraying Abraham. Rather we might say, the space in Velázquez's painting is vague to such a great degree, that this vagueness hinders in our ability to define our position with respect to this space.

Of course, baroque painters do not use these principles in isolation. They combine them in order to achieve a more complex result, until we are unable to identify precisely which principle creates what effect. For instance Caravaggio in his *David with the head of Goliath*, beside the dark background, also uses a gesture: David is depicted as if he were slightly leaning forward, passing us the head of Goliath out of the painting, a horrible and yet serene image. The painting also contains some oblique lines, among which the most distinct is the line of the clothes, which cover a part of David's naked body.

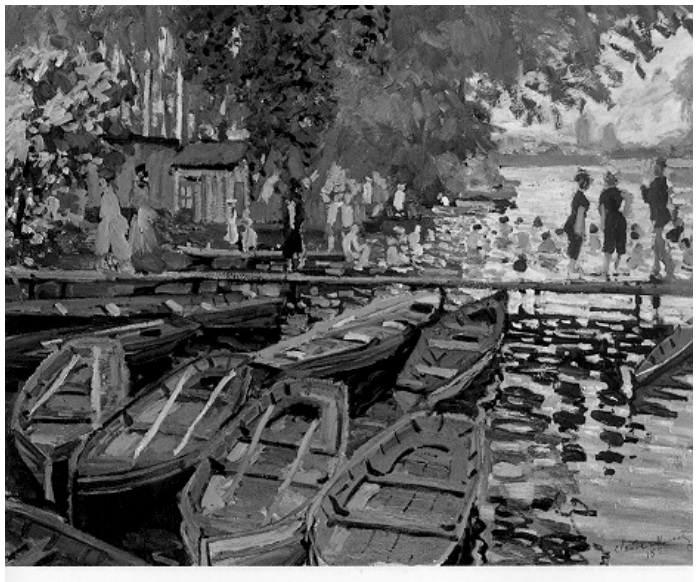
4. The transition to the integrative form: Turner, the impressionists, and Seurat

The aim of the present chapter is to argue, that the Impressionist painters expressed in their paintings ideas, analogous of to the geometric structures that can be found in the works of Arthur Cayley and Felix Klein. It is well known that impressionists tried to paint their momentary visual impressions. Thus instead of permanent objects they tried to capture the evanescent moment. The English painter Joseph Turner (1775-1851) was in many respects a forerunner of Impressionism.

In his painting *Rain, Steam, Speed* (1842) Turner tries to capture the impression of the ephemeral moment. He totally neglects the details and wants to evoke the immediate feeling of the motion of the locomotive crushing its way through the rain. The reality of objects and of the landscape is reduced to a minimum, what remains is light and color. The painting represents the outcome of a long process of development, in which Turner step by step liberated his paintings from realism and opened them to light. We believe, that this shift of Turner is parallel to the geometric ideas of Arthur Cayley, who in a similar way liberated geometry from the "realism" of the Euclidean structure and founded geometry on a deeper structure of the projective space. Turner in a similar manner abandoned the representation of the realistic structure of objects and searched for a neutral basis, on which the realistic structure was built. In Turner this neutral basis was light, which by its rays forms a projective structure, and that projective structure was chosen as the neutral basis of

geometry by Cayley.

These ideas obtained a clearer expression and a more precise technical manifestation in the works of the Impressionist painters. Let us take for instance the painting, *Bathers at La Grenouillere* (1869) from Calude Monet (1840-1926). A rather big portion of the painting is covered by the surface of water. "*The Impressionists often painted water because its amorphous, unarticulated form is ideally suited for representing movement and the momentary, flickering reflections of light and color. Impressionist painters focused exclusively on making a coherent statement based on color and light, independent of meaning. They sought to capture the first impression without the inference of intellectual elements; it was an attempt to represent reality as it was experienced, not as it was known to be*" (Blatt 1984, p. 295).



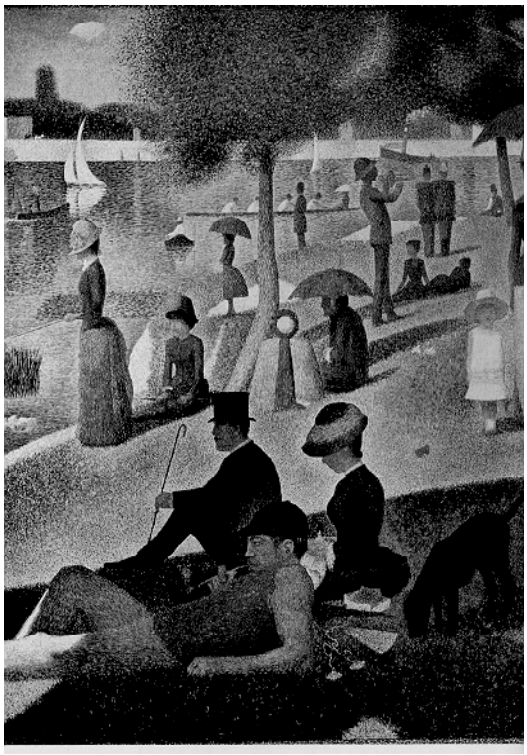
When we look at a person or a thing, we automatically pass from the visual perception to its meaning, the perceived object. The Impressionists wanted to suppress this semantic level of seeing in order to let the perceptual level manifest itself. They wanted to learn to see the word not as something well known and familiar to them, but on the contrary, as if they would see it for the first time. In this respect the surface of water is an ideal object, because its forms have no meaning, and the impressionists tried to see the world so, as if all the figures, hills, buildings and things would be only occasional waves on the rippled surface of reality.

But what is the very first structure, by help of which we interpret our visual impression? It is not difficult to realize, that it is the geometric structure of space. Before starting to identify the particular objects in our visual field, it is necessary to stratify the visual impressions and subject them to spatial forms. Thus before we recognize, that we are looking at a footbridge with several persons standing on it, we must separate the bridge (preliminarily as a neutral and unspecific something) from the background. In this way we introduce into the visual field a fundamental ***geometric*** distinction. We do not perceive the space itself, because the fact that something is far away is, strictly speaking, impossible to perceive. We can only conjecture it, derive it from the

relations between the particular aspects of the visual perception. The spatial structure is thus one of the first structures, by help of which we automatically and unwittingly order and interpret the visual impressions. The visual impressions as such are not spatial; they only obtain a spatial structure in the process of interpretation. What we really see is only light, color, brightness. Everything else is the work of intellect. It is our intellect, which combines the color impressions into objects, it is our intellect, which recognizes, what are we looking at. That there is a bridge on the picture, strictly speaking, cannot be seen, it can only be known. The identification of the bridge is the result of the interpretation of our impressions. A man, grown up in a desert, who newer saw a bridge, could not make that interpretative step, even if he had the same visual impression as we do.

Monet, resembling Turner, tries to destroy the interpretative level of our visual experience in order to reach the level of immediate perceptions, which lies beneath it. In order to capture our immediate perceptions, he had first of all to ***neutralize the spatial structure***. Nevertheless, he did it by more subtle means than Turner did. Turner veiled the whole picture into a vague haze, and so he impeded the spontaneous and automatic identification of the spatial structure of the scene. Monet created techniques, which made it possible, at least for a short time, to get rid of all the interpretative schemes. In contrast to Turner, who impedes the interpretation schemes by veiling the picture into a haze, Monet achieves his goal by the subtle interplay of light on the surface of water. Monet's painting enables us to liberate ourselves, at least for short moments, from the dictate of geometry, from the grip of the interpretative schemes of our intellect. It is sufficient to let our gaze wander for a while on the waves in the lower part of the painting, so that our eye gets used to their changing rhythms. If we then quickly shift our view to the center of the painting, instead of the three figures we will see for a short moment three big dark blots. The eye gets stuck for a while on the level of the purely visual perception. Thus Monet enables us for a short moment to see with our eye and not with our intellect.

In this way Monet succeeded to paint, what the mathematician Arthur Cayley wanted to see. Cayley wanted to pass from the Euclidean plane to the purely projective plane. In connection with Cayley we have written: "*But how did Cayley accomplish this transition? By an appeal! He did not change anything in Beltrami's picture, rather he just asked us to forget that it was drawn on the Euclidean plane and instead to see the whole picture as drawn on the projective plane. That means, we must forget the parallels, the distances and the magnitudes of angles. But who can do this? ... Of course, strictly speaking, we are unable to see this way, but we understand what he wants to tell us. If we are willing to follow him, we are then able to understand the fundamental question of what constitutes the Euclidean structure of Euclidean geometry just as depth was revealed within the Rennaisance painting and just as we got an insight into the non-Euclidean world through the pictures of Lobachevski* " (Kvasz 1998, p. 158). If we leave geometry, we can find a man, who was able, at least for short moments, to pass from the Euclidean plane to its deeper projective structure. That man was Claude Monet, and thanks to his paintings everyone can make that experience.



Similarly, to Felix Klein who created a new geometric language, in which the intuitions of Cayley got a

clear and unambiguous theoretical formulation, in the history of painting it was Georges Seurat (1859-1891), who created a theoretical system, representing the achievements of the impressionists. With his pointillism he created a system, which was the embodiment of the paradigm of the integrative form of language. As we have already mentioned in our paper on the development of geometry, the integrative form of language consists in creating a neutral basis (the projective plane or the projective group) and every geometry–Euclidean, non-Euclidean or affine–can be interpreted as a structure introduced into that neutral basis. Cayley introduced these structures by help of an absolute; Klein used the subgroups of the projective group. We believe, that a similar pattern can be found in the paintings of Seurat.

Let us have a look on a detail of his *Sunday Afternoon on the Island of La Grande Jatte* (1885). We see, that Seurat did not paint figures, trees, or water surface. He painted simply points. On his painting there are no lines, no contours, no areas, the color of which would imitate the color of some object. On the painting there are only points, which form the ***neutral basis*** of his pictorial language, characteristic for the integrative form. The objects arise as ***structures***, introduced into this neutral basis by help of contrasts of colors. Seurat used only points of basic colors and instead of blending colors he achieved the different shades of colors by changing density of the points of the particular basic colors. The basic colors are a kind of generators of the structure of contrasts. Of course the colors do not form a group, therefore the analogy with Klein's approach to geometry is not perfect. Nevertheless, the colors in the paintings of Seurat form another algebraic structure, a three-dimensional vector space; and similarly to Klein's *Erlanger program*, where geometry is defined as a system of invariants of a transformations group, the structure of Seurat's paintings is determined by a system of relations in this space of colors.

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